Circuits

- Kirchoff's rules
- Basic circuit elements
  - R, L, C
  - Units and time scales
  - Addition Rules
- Differential equation framework
- Common Circuit Types
- AC Circuits
  - Inductance
  - RLC circuit equation and solutions
- Other topics
  - Logic Gates

Kirchoff's Laws

- Conservation of Charge ⇔ No net current flux at any point

$$\sum_{i} \vec{I_i} = 0$$

- Conservation of Energy ⇔ No net voltage over any loop

$$\sum_{Loop} V_i = 0$$

- Ohm's law
  - Most general: $\vec{J} = \sigma \vec{E}$
  - More useful for circuits:

$V = IR$ $\quad\quad$ $V = IZ$

Basic Circuit Elements

- Remember how they affect voltages

$$V_R = IR$$

$$V_C = \frac{Q}{C} = \frac{1}{c}\int_0^t I(t')dt$$

$$V_L = L\frac{dI}{dt}$$

Basic Circuit Elements

- Voltage drops ⇔ units
  - [R] = [V/I] = JS/C^2
  - [C] = C/[V] = C^2/J
  - [L] = [V/(I/S)] = JS^2/C^2
- Time scales
  - [RC] = S
  - [L/R] = S
  - [LC] = S^2
- (Will be useful for circuits with time constants)

$$V_R = IR$$

$$V_C = \frac{Q}{C} = \frac{1}{c} \int_0^t I(t')\,dt$$

$$V_L = L\frac{dI}{dt}$$

Addition Rules

- R: Add in series

$$R_{total} = \sum_i R_i$$

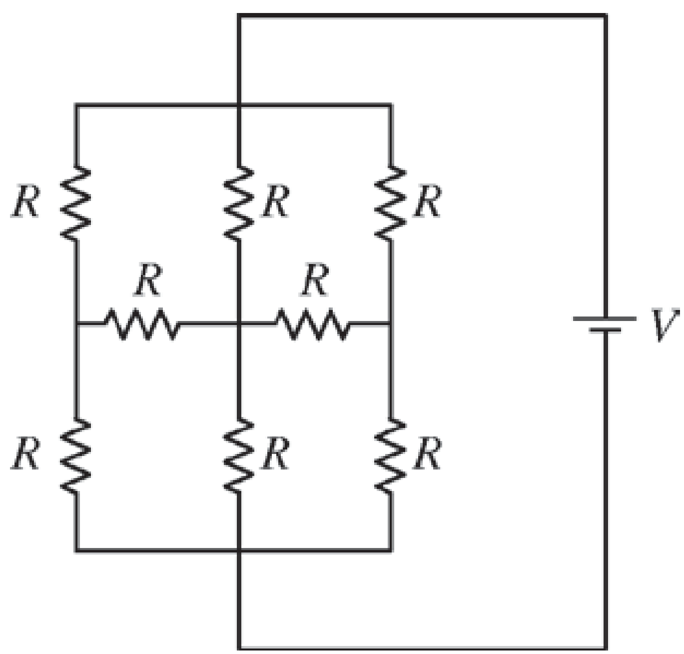
- C: Add in parallel

$$\frac{1}{C_{total}} = \sum_i \frac{1}{C_i}$$

- L: Add in series

$$L_{total} = \sum_i L_i$$

Example



68. The circuit shown in the figure above consists of eight resistors, each with resistance R, and a battery with terminal voltage V and negligible internal resistance. What is the current flowing through the battery?

(A) $\frac{1}{3}\frac{V}{R}$

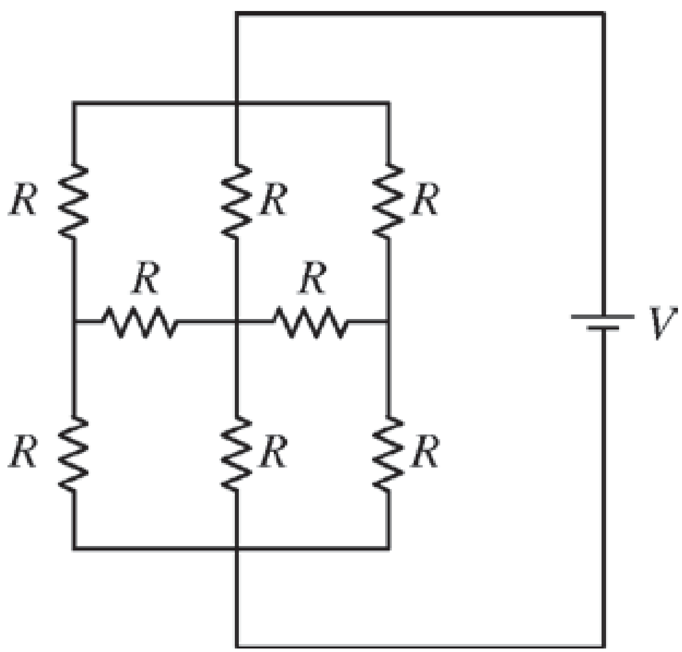
(B) $\frac{1}{2}\frac{V}{R}$

(C) $\frac{V}{R}$

(D) $\frac{3}{2}\frac{V}{R}$

(E) $3\frac{V}{R}$

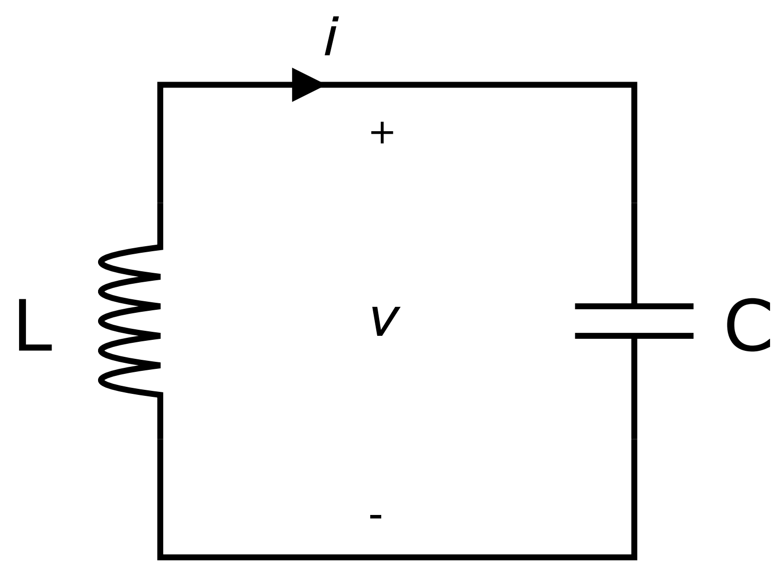
Example



- What is the voltage drop across *each* of the resistors in parallel?
- What does this tell us about the voltage drop across the two center resistors
- (Related problem: symmetric resistor cube)

Differential Equation Framework

- For a single loop in a circuit, we can use Kirchoff's loop rule to write an ODE relating voltage drops across each element
- Example: LC Series Circuit



https://upload.wikimedia.org/wikipedia/commons/b/b2/LC_parallel_simple.svg

Differential Equation Framework

- Voltage drop across capacitor vs time:

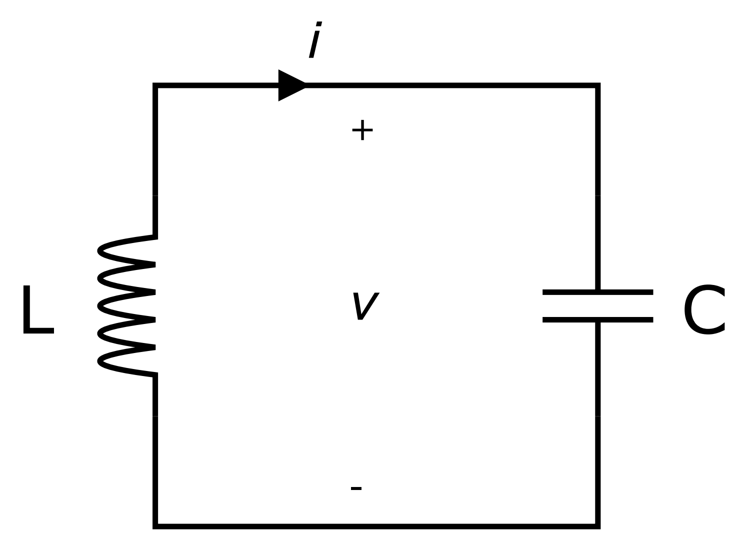
$$V_C = Q(t)/C$$

- Voltage drop across inductor vs time:

$$V_L = L\frac{dI}{dt} = L\frac{d^2Q}{dt^2}$$

- Combine using loop rule:

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0$$



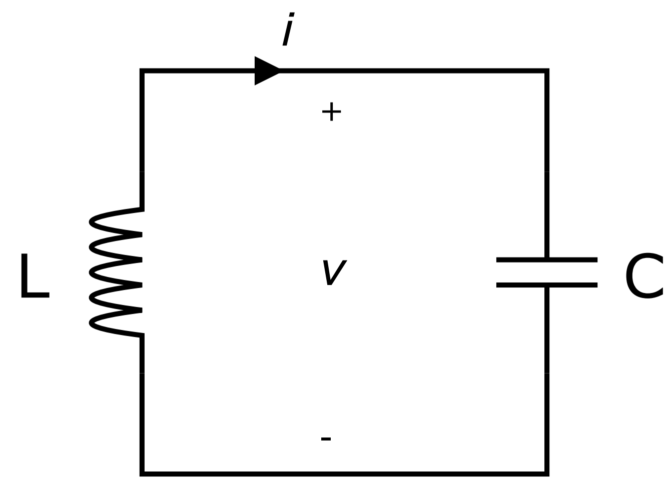
LC Circuit ODE Example

59. For an inductor and capacitor connected in series, the equation describing the motion of charge is

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0,$$

where L is the inductance, C is the capacitance, and Q is the charge. An analogous equation can be written for a simple harmonic oscillator with position x, mass m, and spring constant k. Which of the following correctly lists the mechanical analogs of L, C, and Q?

	L	C	Q
(A)	m	k	x
(B)	m	$1/k$	x
(C)	k	x	m
(D)	$1/k$	$1/m$	x
(E)	x	$1/k$	$1/m$



LR Circuit Example

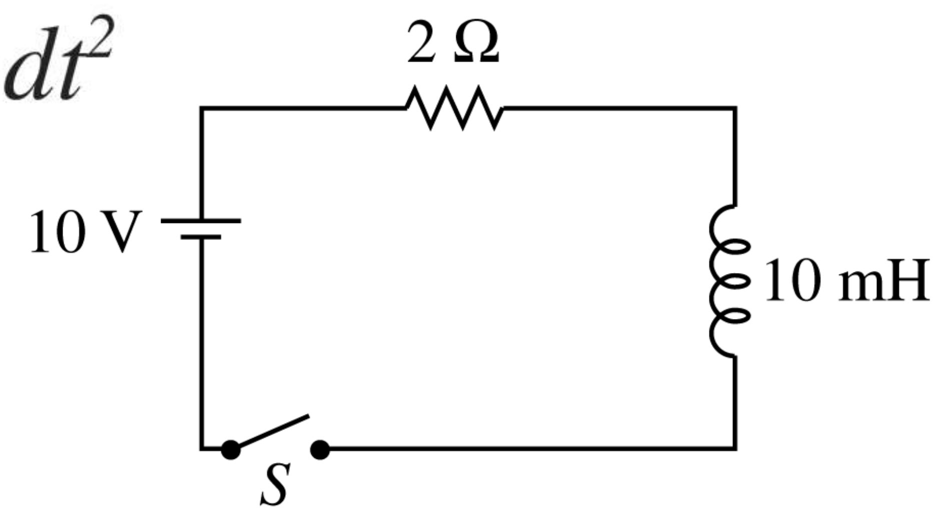
- (Ignore numerical values for *R, L* for now)
- What is the voltage drop across *R*?

$$V_R = RI(t)$$

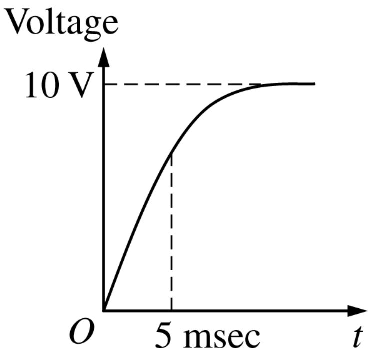
- What is the voltage drop across *L*?

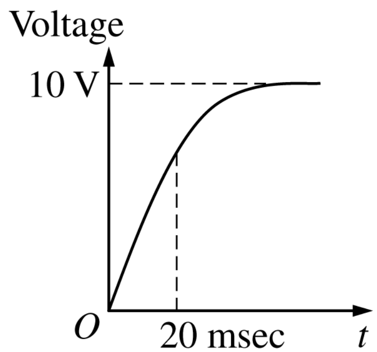
$$V_L = L\frac{d^2I}{dt^2}$$

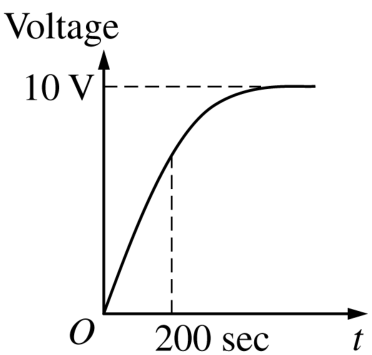
$$V = RI + L\frac{dI}{dt}$$

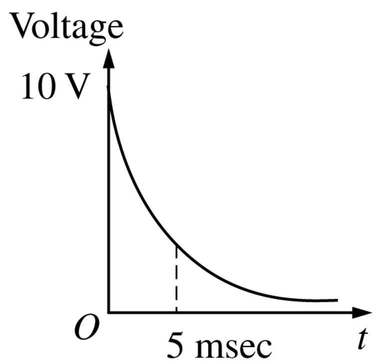


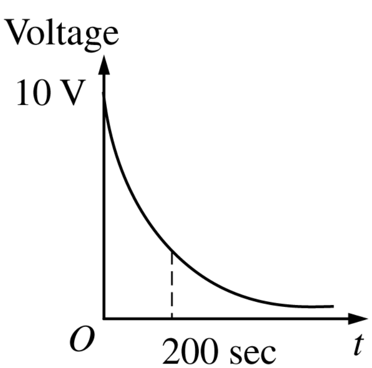
40. In the circuit shown above, the switch S is closed at $t = 0$. Which of the following best represents the voltage across the inductor, as seen on an oscilloscope?

(A)


(B)


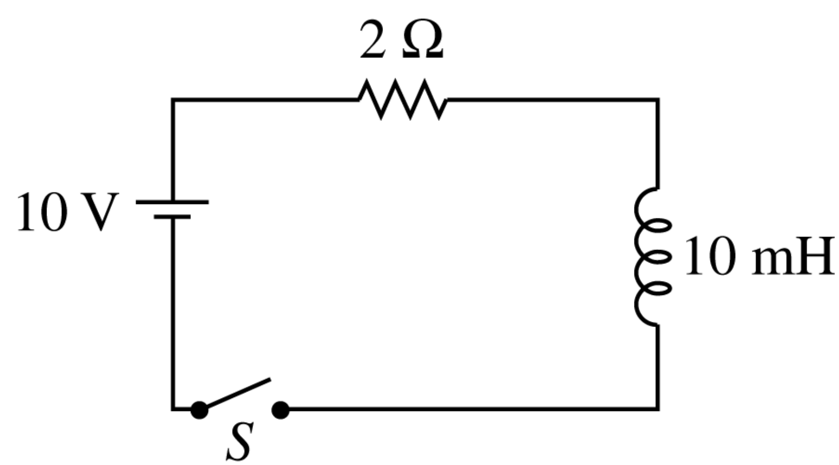
(C)


(D)


(E)


Time scale of solution?

$$V = RI + L\frac{dI}{dt}$$



RLC Circuits

$$\frac{Q}{C}+R\frac{dQ}{dt}+L\frac{d^2Q}{dt^2}=0$$

- Most general solution gives us damped oscillations

$$Q=Q_0e^{-Rt/2L}\cos\left(\left(\sqrt{\frac{1}{LC}+\left(\frac{R}{2L}\right)^2}\right)t-\phi\right)$$

- Important to pay attention to boundary conditions
- Can remove terms based on other circuits

$$L=0 \quad \Rightarrow\frac{Q}{C}+R\frac{dQ}{dt}=0 \qquad C=0 \quad \Rightarrow R\frac{dQ}{dt}+L\frac{d^2Q}{dt^2}=RI+L\frac{dI}{dt}=0$$

Impedance

- Like resistance, but with a phase factor

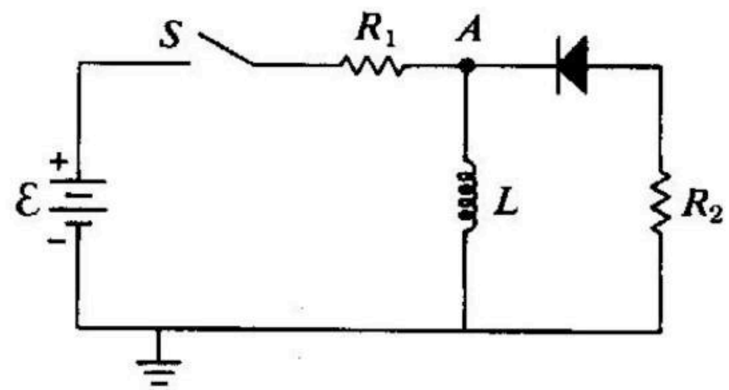
$$V = I X_{RLC}$$

$$X_{RLC} = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

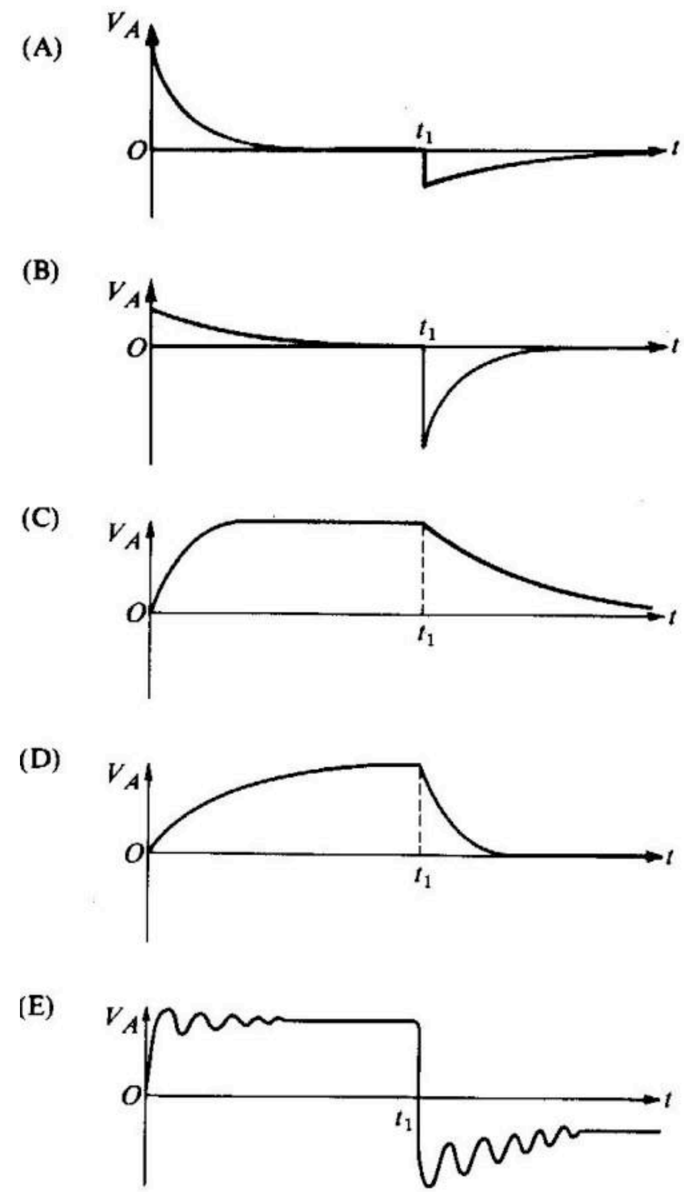
- Affects signal amplitude

Another Example

- What kind of behavior do circuit elements allow?



94. In the circuit shown above, $R_2 = 3R_1$ and the battery of emf ℰ has negligible internal resistance. The resistance of the diode when it allows current to pass through it is also negligible. At time $t = 0$, the switch S is closed and the currents and voltages are allowed to reach their asymptotic values. Then at time t_1, the switch is opened. Which of the following curves most nearly represents the potential at point A as a function of time t?



Another Example

- What is the starting charge on the capacitor?

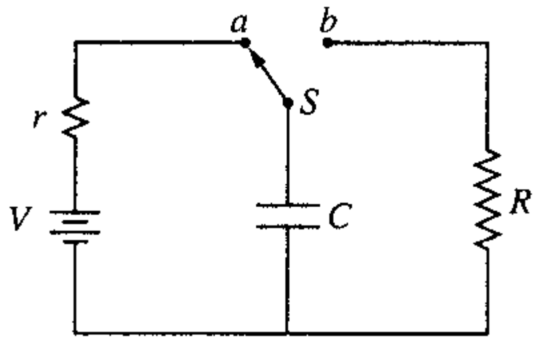


Figure 1

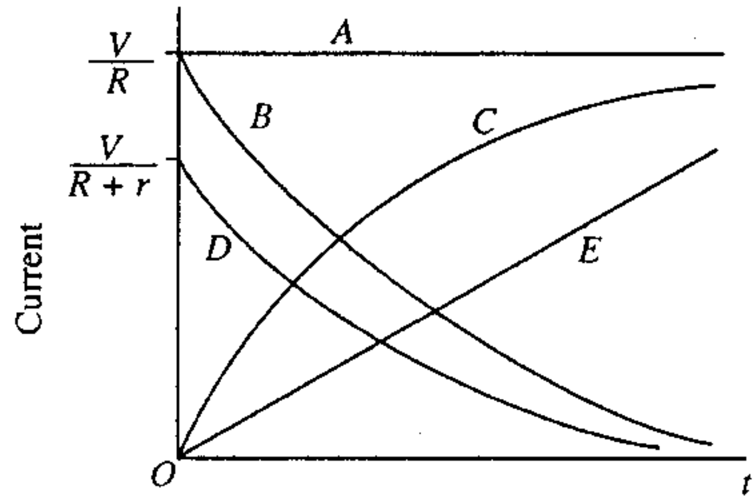


Figure 2

1. The capacitor shown in Figure 1 above is charged by connecting switch S to contact a. If switch S is thrown to contact b at time $t = 0$, which of the curves in Figure 2 above represents the magnitude of the current through the resistor R as a function of time?

   (A) A
   (B) B
   (C) C
   (D) D
   (E) E